ON THE THEORY OF IMPULSIVE FOLLOW-UP SYSTEMS

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1. The equations of motion of an impulsive follow-up system, for which the control signal vanishes during the pause, can be represented for sufficiently small values of the time constants of the control circuits in the following form:

$$\dot{y}_1 - y_2 = 0, \qquad \dot{y}_2 + 2\varepsilon y_2 = \mu k^2 [x(t) - y_1 + q(t)] \tag{1.1}$$

Here

$$\mu = \begin{cases} 1 & \text{for } \vartheta\tau < t < \vartheta\tau + \tau_1 \\ 0 & \text{for } \vartheta\tau + \tau_1 < t < (\vartheta + 1)\tau \end{cases} \qquad \left(\vartheta = \left[\frac{t}{\tau}\right]\right) \tag{1.2}$$

y_1 is the generalized coordinate of the follow-up system, $x(t)$ is the law of motion which the follow-up system must reproduce, τ is the period of alternation, τ_1 is the working interval, $\tau_2 = \tau - \tau_1$ is the pause, $q(t)$ is the additional signal to be given at the entry of the follow-up system for its accelerated adjustment, and ϑ is the integral part of t/τ.

Consider the problem [1] of selecting the law of variation of the function $q(t)$ with respect to the time t in such a way that at the instant $t = T_1$ the adjustment of the follow-up system would occur, i.e. the relations

$$y_1(T_1) = 0, \qquad y_2(T_1) = 0 \tag{1.3}$$

would hold.

We shall assume that $x(t) \equiv 0$ during the time of adjustment and that $q(t)$ is a step-function which preserves its values in time intervals which are multiples of the alternating period τ.

In order to investigate the motion of the follow-up system under consideration it is appropriate to pass from the system of differential equations (1.1) to a system of difference equations. The latter can be obtained by connecting the values y_1 and y_2 at the end and the beginning of one period of alternation. So, for the first period of alternation during the working interval $0 \leqslant t \leqslant \tau_1$ the differential equations

$$\dot{y}_1 - y_2 = 0, \qquad \dot{y}_2 + 2\varepsilon y_2 + k^2 y_1 = k^2 q(0) \tag{1.4}$$

hold.

Corresponding to Equations (1.4) the law of motion of the system during the working interval will be the following:

$$\begin{aligned} y_1(t) &= \frac{1}{\omega}[y_2(0) + \varepsilon y_1(0) - \varepsilon q(0)]\, e^{-\varepsilon t} \sin \omega t + \\ &\quad + [y_1(0) - q(0)]\, e^{-\varepsilon t} \cos \omega t + q(0), \qquad \omega = \sqrt{k^2 - \varepsilon^2} \\ y_2(t) &= y_2(0)\, e^{-\varepsilon t} \cos \omega t + \left[\frac{k^2}{\omega} q(0) - \frac{k^2}{\omega} y_1(0) - \frac{\varepsilon}{\omega} y_2(0)\right] e^{-\varepsilon t} \sin \omega t \qquad (0 \leqslant t \leqslant \tau_1) \end{aligned} \tag{1.5}$$

At the end of the working interval the functions y_1 and y_2 will assume the following values:

$$\begin{aligned} y_1(\tau_1) &= \left(\frac{\varepsilon\nu_1}{\omega} + \nu_2\right) y_1(0) + \frac{\nu_1}{\omega} y_2(0) + \left(1 - \frac{\nu_1 \varepsilon}{\omega} - \nu_2\right) q(0) \\ y_2(\tau_1) &= -\frac{\nu_1 k^2}{\omega} y_1(0) + \left(\nu_2 - \frac{\varepsilon\nu_1}{\omega}\right) y_2(0) + \frac{\nu_1 k^2}{\omega} q(0) \end{aligned} \tag{1.6}$$

where

$$\nu_1 = e^{-\varepsilon\tau_1} \sin \omega\tau_1, \qquad \nu_2 = e^{-\varepsilon\tau_1} \cos \omega\tau_1 \tag{1.7}$$

During the pause $\tau_1 \leqslant t \leqslant \tau$ the differential equations of motion according to (1.1) will have the form

$$\dot{y}_1 - y_2 = 0, \qquad \dot{y}_2 + 2\varepsilon y_2 = 0 \tag{1.8}$$

The law of motion of the system during the pause will be the following:

$$y_1(t) = y_1(\tau_1) + \frac{1}{2\varepsilon} y_2(\tau_1)\,[1 - e^{-2\varepsilon(t-\tau_1)}], \quad y_2(t) = y_2(\tau_1)\, e^{-2\varepsilon(t-\tau_1)} \quad (\tau_1 \leqslant t \leqslant \tau) \tag{1.9}$$

At the end of the pause, i.e. at the instant $t = \tau$, the functions y_1 and y_2 will have the following values:

$$y_1(\tau) = y_1(\tau_1) + \frac{1-\nu_3}{2\varepsilon} y_2(\tau_1), \quad y_2(\tau) = \nu_3 y_2(\tau_1), \qquad \nu_3 = e^{-2\varepsilon\tau_2} \tag{1.10}$$

Substituting into the expressions (1.10) the values $y_1(\tau_1)$ and $y_2(\tau_1)$ from (1.6) we obtain

$$\begin{aligned} y_1(\tau) &= -a_{11} y_1(0) - a_{12} y_2(0) + (1 + a_{11})\, q(0) \\ y_2(\tau) &= -a_{21} y_1(0) - a_{22} y_2(0) + a_{21}\, q(0) \end{aligned} \tag{1.11}$$

where

$$a_{11} = -\left(\frac{\varepsilon\nu_1}{\omega} + \nu_2 - \frac{\nu_1 k^2}{\omega}\frac{1-\nu_3}{2\varepsilon}\right), \qquad a_{21} = \frac{\nu_1\nu_3 k^2}{\omega}$$
$$a_{12} = -\left[\frac{\nu_1}{\omega} + \frac{1-\nu_3}{2\varepsilon}\left(\nu_2 - \frac{\varepsilon\nu_1}{\omega}\right)\right], \qquad a_{22} = -\nu_3\left(\nu_2 - \frac{\varepsilon\nu_1}{\omega}\right) \tag{1.12}$$

The relations (1.11) connect the values of the functions y_1 and y_2 at the end and the beginning of the first period of alternation. Obviously, analogous relations will hold for any (nth) period of alternation

$$\begin{aligned} y_1((n+1)\tau) + a_{11}y_1(n\tau) + a_{12}y_2(n\tau) &= (1+a_{11})\,q(n\tau) \\ y_2((n+1)\tau) + a_{21}y_1(n\tau) + a_{22}y_2(n\tau) &= a_{21}\,q(n\tau) \end{aligned} \tag{1.13}$$

Equations (1.13) represent the difference equations which describe the motion of the considered impulsive follow-up system

Introducing the matrices

$$f(T) = \begin{Vmatrix} T + a_{11} & a_{12} \\ a_{21} & T + a_{22} \end{Vmatrix}, \qquad y = \begin{Vmatrix} y_1 \\ y_2 \end{Vmatrix}, \qquad b = \begin{Vmatrix} 1 + a_{11} \\ a_{21} \end{Vmatrix} \tag{1.14}$$

where T is the anticipation operator determined by the relation

$$T^s y_k = y_k(t + s\tau)$$

we obtain the matrix difference equation

$$f(T)\,y(t) = bq(t) \tag{1.15}$$

which is equivalent to the system of scalar difference equations (1.13).

Assume that the elements y_1 and y_2 of the matrix y in the time interval $0 < t < \tau$ coincide with $y_1^*(t)$ and $y_2^*(t)$ determined by the expressions (1.5) and (1.9):

$$y(t) = y^*(t) \qquad (0 \leqslant t \leqslant \tau) \tag{1.16}$$

Under these conditions the solution of the matrix difference equation (1.15) can be constructed by means of the methods of the operational calculus. Letting

$$\xi(p) \doteqdot q(t), \qquad \eta(p) \doteqdot y(t) \tag{1.17}$$

and taking into account that

$$Ty_i \doteqdot e^{p\tau}[\eta_i(p) - \alpha_i(p)], \quad \alpha_i(p) = p\int_0^\tau y_i^*(t)\,e^{-pt}\,dt \qquad (i = 1, 2) \tag{1.18}$$

we obtain for the matrix equation (1.16) the following image equation:

$$f(\gamma)\,\eta(p) - \gamma\alpha(p) = b\xi(p) \tag{1.19}$$

where

$$\gamma = e^{p\tau}, \qquad \eta(p) = \begin{Vmatrix} \eta_1(p) \\ \eta_2(p) \end{Vmatrix}, \qquad \alpha(p) = \begin{Vmatrix} \alpha_1(p) \\ \alpha_2(p) \end{Vmatrix} \tag{1.20}$$

From Equation (1.19) we find

$$\eta(p) = \gamma\frac{F(\gamma)\,\alpha(p)}{\Delta(\gamma)} + \frac{F(\gamma)\,b}{\Delta(\gamma)}\xi(p) \tag{1.21}$$

where $F(\gamma)$ is the adjoint matrix of the matrix $f(\gamma)$:

$$F(\gamma) = \begin{Vmatrix} \gamma + a_{22} & -a_{12} \\ -a_{21} & \gamma + a_{11} \end{Vmatrix}$$

and $\Delta(\gamma)$ is the determinant of the matrix $f(\gamma)$:

$$\Delta(\gamma) = \gamma^2 + (a_{11} + a_{22})\gamma + a_{11}a_{22} - a_{12}a_{21} = (\gamma - \gamma_1)(\gamma - \gamma_2) \tag{1.22}$$

Denote by $M(t)$ and $L(t)$ the originals of the following images:

$$\gamma\frac{F(\gamma)\,\alpha(p)}{\Delta(\gamma)} \doteqdot M(t), \qquad (\gamma - 1)\frac{F(\gamma)\,b}{\Delta(\gamma)} \doteqdot L(t) \tag{1.23}$$

Since

$$\gamma\frac{F(\gamma)\,\alpha(p)}{\Delta(\gamma)} = \frac{F(\gamma_1)}{\gamma_1 - \gamma_2}\frac{\gamma\alpha(p)}{\gamma - \gamma_1} + \frac{F(\gamma_2)}{\gamma_2 - \gamma_1}\frac{\gamma\alpha(p)}{\gamma - \gamma_2} \tag{1.24}$$

$$\frac{\gamma\alpha(p)}{\gamma - \gamma_i} \doteqdot y^*(t - \vartheta\tau)\,\gamma_i^{\vartheta} \tag{1.25}$$

where $y^*(t - \vartheta\ \tau)$ is a periodic function of period τ, then

$$M(t) = \left[\frac{F(\gamma_1)}{\gamma_1 - \gamma_2}\gamma_1^{\vartheta} + \frac{F(\gamma_2)}{\gamma_2 - \gamma_1}\gamma_2\right] y^*(t - \vartheta\tau) \tag{1.26}$$

Analogously

$$(\gamma - 1)\frac{F(\gamma)\,b}{\Delta(\gamma)} = \frac{F(\gamma_1)\,b}{\gamma_1 - \gamma_2}\frac{\gamma - 1}{\gamma - \gamma_1} + \frac{F(\gamma_2)\,b}{\gamma_2 - \gamma_1}\frac{\gamma - 1}{\gamma - \gamma_2} \tag{1.27}$$

$$L(t) = \frac{F(\gamma_1)\,b}{\gamma_1 - \gamma_2}\gamma_1^{\vartheta} + \frac{F(\gamma_2)\,b}{\gamma_2 - \gamma_1}\gamma_2^{\vartheta} \tag{1.28}$$

As seen from (1.28), $L(t)$ is a step-function. According to the assumption made above the function $q(t)$ is also a step-function. Then, on the basis of a theorem on the multiplication of the images of step-functions, we obtain

$$\frac{F(\gamma)\,b}{\Delta(\gamma)}\xi(p) \doteqdot \sum_{j=1}^{\vartheta} L(\vartheta\tau - j\tau)\,q(j\tau - \tau) \tag{1.29}$$

Thus, the solution of the matrix difference equation (1.15), satisfying the condition (1.16), has the following form:

$$y(t)=\left[\frac{F(\gamma_1)}{\gamma_1-\gamma_2}\gamma_1^{\vartheta}+\frac{F(\gamma_2)}{\gamma_2-\gamma_1}\gamma_2^{\vartheta}\right]y^*(t-\vartheta\tau)+\sum_{j=1}^{\vartheta}L(\vartheta\tau-j\tau)\,q(j\tau-\tau) \quad (1.30)$$

The elements of the matrix $y(t)$ are

$$y_i(t)=\left[\frac{F_{i1}(\gamma_1)}{\gamma_1-\gamma_2}y_1^*(t-\vartheta\tau)+\frac{F_{i2}(\gamma_1)}{\gamma_1-\gamma_2}y_2^*(t-\vartheta\tau)\right]\gamma_1^{\vartheta}+\left[\frac{F_{i1}(\gamma_2)}{\gamma_2-\gamma_1}y_1^*(t-\vartheta\tau)+\right.$$

$$\left.+\frac{F_{i2}(\gamma_2)}{\gamma_2-\gamma_1}y_2^*(t-\vartheta\tau)\right]\gamma_2^{\vartheta}+\sum_{j=1}^{\vartheta}L_i(\vartheta\tau-j\tau)\,q(j\tau-\tau) \qquad (i=1,2) \quad (1.31)$$

In order that at the instant $t = T_1 = \vartheta_1\tau$ the follow-up system be adjusted, i.e. the relations (1.3)

$$y_1(T_1)=0, \qquad y_2(T_1)=0$$

hold, the following conditions which can be obtained by means of (1.31),

$$\sum_{j=1}^{\vartheta_1}L_i(\vartheta_i\tau-j\tau)\,q(j\tau-\tau)=R_i(T_1) \qquad (i=1,2) \qquad (1.32)$$

must hold, where

$$R_i(T_1)=-\left[\frac{F_{i1}(\gamma_1)}{\gamma_1-\gamma_2}y_1^*(0)+\frac{F_{i2}(\gamma_1)}{\gamma_1-\gamma_2}y_2^*(0)\right]\gamma_1^{\vartheta_1}-$$

$$-\left[\frac{F_{i1}(\gamma_2)}{\gamma_2-\gamma_1}y_1^*(0)+\frac{F_{i2}(\gamma_2)}{\gamma_2-\gamma_1}y_2^*(0)\right]\gamma_2^{\vartheta_1} \quad (i=1,2) \quad (1.33)$$

Decompose the interval $(0, T_1)$ into two intervals $(0, j_1\tau)$ and $(j_1\tau, T_1)$ and assume that the function $q(t)$ is a step-function which preserves its values in these time intervals. Denote these values by $q(0)$ and $q(t_1)$, respectively. The relations (1.32) then assume the form

$$c_i^{(0)}q(0)+c_i^{(1)}q(t_1)=R_i(T_1) \qquad (i=1,\ 2) \qquad (1.34)$$

where

$$c_i^{(0)}=\sum_{j=1}^{j_1}L_i(\vartheta_1\tau-j\tau), \qquad c_i^{(1)}=\sum_{j=j_1+1}^{\vartheta_1}L_i(\vartheta_1\tau-j\tau) \qquad (i=1,\ 2) \qquad (1.35)$$

From Equations (1.35) we obtain

$$q(0)=\frac{\Delta_1}{\Delta}, \qquad q(t_1)=\frac{\Delta_2}{\Delta} \qquad (1.36)$$

where

$$\Delta_1 = \begin{vmatrix} R_1(T_1) & c_1^{(1)} \\ R_2(T_1) & c_2^{(1)} \end{vmatrix}, \qquad \Delta_2 = \begin{vmatrix} c_1^{(0)} & R_1(T_1) \\ c_2^{(0)} & R_2(T_1) \end{vmatrix}, \qquad \Delta = \begin{vmatrix} c_1^{(0)} & c_1^{(1)} \\ c_2^{(0)} & c_2^{(1)} \end{vmatrix} \tag{1.37}$$

The expressions (1.36) determine the law according to which $q(t)$ must be varied in order that at the instant $t = T_1$ the follow-up system be adjusted.

2. Suppose that the strengthening coefficient of the follow-up system varies with time. Then the coefficient k^2 entering into Equations (1.1) will be a certain function of the time

$$k^2 = \varkappa(t) \tag{2.1}$$

We shall assume that $\kappa(t)$ is a step-function, the width of the steps being equal to the period of alternation of the follow-up system.

Then during each separate period of alternation the differential equations (1.4) and (1.8) will have constant coefficients while the parameters ω, ν_1, ν_2, a_{11}, a_{12}, a_{21} and a_{22}, determined by the expressions (1.7) and (1.12), will be certain functions of the time which will be determined provided that the function $\kappa(t)$ is given.

The system of difference equations (1.13) in the given case can be represented by the following matrix difference equation:

$$Ty + a(t)\, y = Q(t) \tag{2.2}$$

where

$$y = \begin{Vmatrix} y_1 \\ y_2 \end{Vmatrix}, \qquad a(t) = \begin{Vmatrix} a_{11}(t) & a_{12}(t) \\ a_{21}(t) & a_{22}(t) \end{Vmatrix}, \qquad Q(t) = \begin{Vmatrix} [1 + a_{11}(t)]\, q(t) \\ a_{21}(t)\, q(t) \end{Vmatrix} \tag{2.3}$$

The solution of Equation (2.2) has the following form:

$$y(t) = \theta(t)\, \theta^{-1}(t - \vartheta\tau)\, y^*(t - \vartheta\tau) + \sum_{j=1}^{\vartheta} \theta(t)\, \theta^{-1}(t - \vartheta\tau + j\tau)\, Q(t - \vartheta\tau + j\tau - \tau) \tag{2.4}$$

where $\theta(t)$ is a square matrix, the columns of which are linearly independent solutions of the homogeneous matrix equation

$$Ty + a(t)\, y = 0 \tag{2.5}$$

The matrix $\theta^{-1}(t)$ is the inverse matrix of $\theta(t)$.

In the expression (2.4) the second term vanishes in the interval $0 < t < \tau$. Therefore, according to (2.4)

$$y(t) = y^*(t) \qquad (0 \leqslant t \leqslant \tau) \tag{2.6}$$

holds, where $y^*(t)$ is a matrix, the elements of which in the interval $0 \leqslant t \leqslant \tau$ are determined by the expressions (1.5) and (1.9).

Denoting by $N(t,\ j\tau)$ a matrix function of weight

$$N(t,\ j\tau) = \theta(t)\,\theta^{-1}(t - \vartheta\tau + j\tau) \tag{2.7}$$

the solution (2.4) can be put in the form

$$y(t) = N(t,\ 0)\,y^*(t - \vartheta\tau) + \sum_{j=1}^{\vartheta} N(t,\ j\tau)\,Q(t - \vartheta\tau + j\tau - \tau) \tag{2.8}$$

The elements of the matrix $y(t)$ according to (2.8) have the form

$$y_i(t) = \sum_{k=1}^{2} N_{ik}(t,\ 0)\,y_k^*(t - \vartheta\tau) + \sum_{k=1}^{2}\sum_{j=1}^{\vartheta} N_{ik}(t,\ j\tau)\,Q_k(t - \vartheta\tau + j\tau - \tau) \quad (i = 1,2) \tag{2.9}$$

Substituting the values Q_k given by (2.3) we can reduce the expressions (2.9) to the following form:

$$y_i(t) = \sum_{k=1}^{2} N_{ik}(t,\ 0)\,y_k^*(t - \vartheta\tau) + \sum_{j=1}^{\vartheta} W_i(t,\ j\tau)\,q(t - \vartheta\tau + j\tau - \tau) \quad (i = 1,2) \tag{2.10}$$

where

$$W_i(t,\ j\tau) = N_{i1}(t,\ j\tau)\,[1 + a_{11}(t - \vartheta\tau + j\tau - \tau)] + N_{i2}(t,\ j\tau)\,a_{21}(t - \vartheta\tau + j\tau - \tau) \quad (i = 1,\ 2) \tag{2.11}$$

At the instant $t = T_1 = \vartheta_1\tau$ the expressions (2.10) assume the form

$$y_i(T_1) = \sum_{k=1}^{2} N_{ik}(T_1,\ 0)\,y_k^*(0) + \sum_{j=1}^{\vartheta_1} W_i(T_1,\ j\tau)\,q(j\tau - \tau) \quad (i = 1,\ 2) \tag{2.12}$$

where according to (2.11) we have

$$W_i(T_1,\ j\tau) = N_{i1}(T\ ,\ j\tau)\,[1 + a_{11}(j\tau - \tau)] + N_{i2}(T_1,\ j\tau)\,a_{21}(j\tau - \tau) \quad (i = 1,\ 2) \tag{2.13}$$

In order that at the instant $t = T_1$ the follow-up system be adjusted, i.e. the relations (1.3),

$$y_1(T_1) = 0, \qquad y_2(T_1) = 0$$

hold, the following conditions must be satisfied:

$$\sum_{j=1}^{\vartheta_1} W_i(T_1,\ j\tau)\,q(j\tau - \tau) = R_i^*(T_1), \quad (i = 1,\ 2) \tag{2.14}$$

where

$$R_i^*(T_1) = -\sum_{k=1}^{2} N_{ik}(T_1,\ 0)\, y_k^*(0) \qquad (2.15)$$

Decomposing as above the time interval $(0,\ T_1)$ into two intervals $(0,\ j_1\tau)$ and $(j_1\tau,\ T_1)$, and assuming $q(t)$ to be a step-function, the values of which in these intervals are $q(0)$ and $q(t_1)$, respectively, we can reduce Equations (2.14) to the form

$$s_i^{(0)} q(0) + s_i^{(1)} q(t_1) = R_i^*(T_1) \qquad (i = 1,\ 2) \qquad (2.16)$$

where

$$s_i^{(0)} = \sum_{j=1}^{j_1} W_i(T_1,\ j\tau), \qquad s_i^{(1)} = \sum_{j=j_1+1}^{\vartheta_1} W_i(T_1,\ j\tau) \qquad (i = 1, 2) \qquad (2.17)$$

Thus, the values of $q(0)$ and $q(t_1)$ will be

$$q(0) = \frac{\Delta_1^*}{\Delta^*}, \qquad q(t_1) = \frac{\Delta_2^*}{\Delta^*} \qquad (2.18)$$

where

$$\Delta_1^* = \begin{vmatrix} R_1^*(T_1) & s_1^{(1)} \\ R_2^*(T_1) & s_2^{(1)} \end{vmatrix}, \qquad \Delta_2^* = \begin{vmatrix} s_1^{(0)} & R_1^*(T_1) \\ s_2^{(0)} & R_2^*(T_1) \end{vmatrix}, \qquad \Delta^* = \begin{vmatrix} s_1^{(0)} & s_1^{(1)} \\ s_2^{(0)} & s_2^{(1)} \end{vmatrix} \qquad (2.19)$$

Calculating the quantities (2.18), the functions $N_{ik}(T_1,\ j\tau)$ $(i,\ k = 1,\ 2)$, occurring in the expressions (2.13) and representing for a fixed value $t = T_1$ the elements of a matrix function of weight $N(t,\ j\tau)$, are assumed to be known in the interval $0 < t < T_1 = \vartheta_1\tau$. Analogously are assumed to be known the quantities $N_{ik}(T_1,\ 0)$ $(i,\ k = 1,\ 2)$, occurring in the expressions (2.15) and representing for $t = T_1$, $j = 0$ the values of the elements of a matrix function of weight $N(t,\ j\tau)$.

From the results obtained in the paper [2] it follows that

$$N_{1k}(T_1,\ j\tau) = Y_k(j\tau) \qquad (k = 1,\ 2) \qquad (2.20)$$

where Y_k are the solutions of the conjugate system of difference equations

$$\begin{aligned} Y_1(t) + a_{11}(t) Y_1(t+\tau) + a_{21}(t) Y_2(t+\tau) &= 0 \\ Y_2(t) + a_{12}(t) Y_1(t+\tau) + a_{22}(t) Y_2(t+\tau) &= 0 \end{aligned} \qquad (2.21)$$

constructed for the system of difference equations (2.2), and satisfying in the interval $\vartheta_1\tau < t < (\vartheta_1 + 1)\tau$ the conditions

$$Y_1(t) = 1, \qquad Y_2(t) = 0 \tag{2.22}$$

Analogously

$$N_{2k}(T_1, j\tau) = Y_k^*(j\tau) \qquad (k = 1, 2) \tag{2.23}$$

where $Y_k^*(j\tau)$ are the solutions of the system of difference equations (2.21), satisfying in the interval $\vartheta_1\tau < t < (\vartheta_1 + 1)\tau$ the conditions

$$Y_1(t) = 0, \qquad Y_2(t) = 1 \tag{2.24}$$

3. As an example, consider an impulsive follow-up system with the following parameters:

$$\varepsilon = 5.275 \text{ sec}^{-1}, \qquad k^2 = 7500 \text{ sec}^{-2}, \qquad \tau_1 = 0.01 \text{ sec}, \qquad \tau_2 = 0.03 \text{ sec}$$

The time interval during which the follow-up system must be adjusted is $T_1 = 4\tau = 0.16$ sec. The initial deviations are $y_1(0) = 0.4$, $y_2(0) = 20$ sec^{-1}.

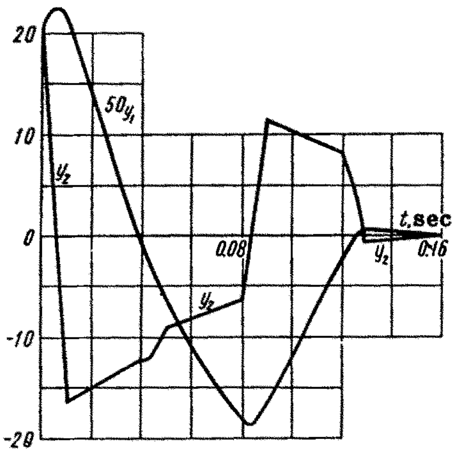

FIG. 1.

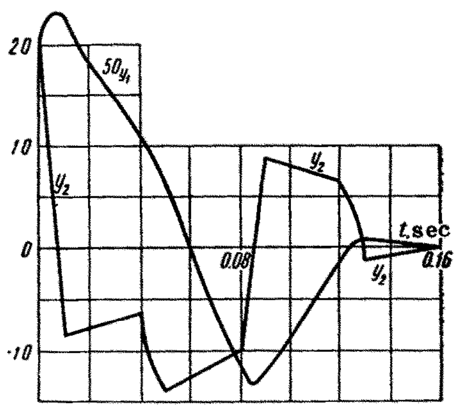

FIG. 2.

For $j_1 = 2$ the values of $q(0)$ and $q(t_1)$ are the following

$$q(0) = -0.0504, \qquad q(t_1) = -0.128$$

The process of adjustment of the follow-up system is represented by the graphs of the functions $y_1(t)$ and $y_2(t)$ in Fig. 1. For the same data but a variable strengthening coefficient

$$k^2 = \varkappa(t) = 7500 + 1000\vartheta$$

the values of $q(0)$ and $q(t_1)$ are the following:

$$q(0) = 0.0768, \qquad q(t_1) = -0.0619$$

The process of adjustment of the follow-up system for a variable strengthening coefficient is represented by the graphs of the functions $y_1(t)$ and $y_2(t)$ in Fig. 2.

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